

# POSSIBLE TIME VARIATIONS OF $G$ IN SCALAR-TENSOR THEORIES OF GRAVITY

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We estimate the possible variations of the gravitational constant  $G$  in the framework of a generalized (Bergmann-Wagoner-Nordtvedt) scalar-tensor theory of gravity on the basis of the field equations, without using their special solutions. Specific estimates are essentially related to the values of other cosmological parameters (the Hubble and acceleration parameters, the dark matter density etc.), but the values of  $\dot{G}/G$  compatible with modern observations do not exceed  $10^{-12}$ .

## 1. Introduction

Dirac's Large Numbers Hypothesis (LNH) is the origin of many theoretical explorations of time-varying  $G$ . According to the LNH, the value of  $\dot{G}/G$  should approximately coincide with the Hubble rate. Although it has become clear in the recent decades that the Hubble rate is too high to be compatible with experiment, the enduring legacy of Dirac's bold stroke is the acceptance by modern theories of non-zero values of  $\dot{G}/G$  as being potentially consistent with physical reality.

There are three problems related to  $G$ , whose origin lies mainly in unified model predictions: 1) absolute  $G$  measurements, 2) possible time variations of  $G$ , 3) possible range variations of  $G$ , i.e., non-Newtonian, or new interactions. For 1) and 3) see [4].

After the original *Dirac hypothesis* some new concepts appeared and also some generalized *theories* of gravitation admitting variations of the effective gravitational coupling. We can single out three stages in the development of this field:

1. Study of theories and hypotheses with variations of fundamental physical constants, their predictions and confrontation with experiments (1937-1977).
2. Creation of theories admitting variations of an effective gravitational constant in a particular system of units, analyses of experimental and observational data within these theories [1] (1977-present).

3. Analyses of variations of fundamental physical constants within unified models [4] (present).

Different theoretical schemes lead to temporal variations of the effective gravitational constant:

1. Empirical models and theories of Dirac type, where  $G$  is simply replaced with  $G(t)$ .
2. Numerous scalar-tensor theories (STT) of Jordan-Brans-Dicke type, with  $G$  depending on the scalar field  $\phi(t)$  or a number of scalar fields.
3. Gravitational theories with a nonminimally coupled (in particular, conformal) scalar field arising in different approaches [1] (they can actually be treated as special cases of STT).
4. Multidimensional unified theories in which there are dilatonic fields and effective scalar fields appearing in our 4-dimensional spacetime from extra dimensions [4]. They may also help one in solving the problem of a variable cosmological constant from Planckian to present values and the cosmic coincidence problem.

A striking feature of the present status of theoretical physics is that there is no satisfactory theory unifying all four known interactions; most modern unification theories do not admit unique and universal constant values of physical constants and of the Newtonian gravitational coupling constant  $G$  in particular. In this paper we discuss the bounds that may be suggested by a general class of STT. One can mention that STT are among the viable alternatives to general relativity; on the one hand, they are widely used for comparison with observations and, on the other, their different versions emerge in the field limits of the candidate "theories of everything".

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Although the bounds on  $\dot{G}$  and  $G(r)$  are in some classes of theories rather wide on purely theoretical grounds, since any theoretical model contains a number of adjustable parameters, we note that observational data concerning other phenomena, in particular, cosmological data, place limits on the possible ranges of these adjustable parameters.

Here we restrict ourselves to the problem of  $\dot{G}$  (for  $G(r)$  see [1–4]). We show that various theories predict the value of  $\dot{G}/G$  to be  $10^{-12}/\text{yr}$  or less. The significance of this fact for experimental and observational determinations of the value of or upper bound on  $\dot{G}$  is the following: any determination with error bounds significantly better than  $10^{-12}/\text{yr}$  (combined with experimental bounds on other parameters) will typically be compatible with only a small portion of existing theoretical models and will therefore cast serious doubt on the viability of all other models. In short, a tight bound on  $\dot{G}$ , in conjunction with other astrophysical observations, will be a very effective “theory killer” and/or significantly reduce the class of viable theories. Any step forward in this direction will be of utmost significance.

Some estimations of  $\dot{G}$  had been done long ago in the framework of general scalar-tensor and multidimensional theories using the values of cosmological parameters ( $\Omega$ ,  $H$ ,  $q$  etc) known at that time [1, 4, 5, 6]. It is easy to show that for modern values they predict  $\dot{G}/G$  at the level of  $10^{-12}/\text{yr}$  and less (see also recent estimations of A. Miyazaki [7], predicting time variations of  $G$  at the level of  $10^{-13}\text{yr}^{-1}$  for a Machian-type cosmological solution in the Brans-Dicke theory).

The most reliable, by now, experimental bounds on  $\dot{G}/G$  (spacecraft radar ranging [8]) and lunar laser ranging [9]) give a limit of  $10^{-12}/\text{yr}$ , so any results at this level or better will be very important for solving the fundamental problem of variations of constants and for discriminating between viable unified theories. So, realization of such multipurpose new generation type space experiments like Satellite Energy Exchange (SEE) for measuring  $\dot{G}$  and also absolute value of  $G$  and Yukawa type forces at the ranges of metres and the Earth radius [10, 12] become extremely topical.

## 2. Scalar-tensor cosmology and variations of $G$

We are going to estimate the order of magnitude of variations of the gravitational constant  $G$  due to cosmological expansion in the framework of scalar-tensor theories (STT) of gravity, using modern data on the cosmological parameters.

Consider the general (Bermann-Wagoner-Nordtvedt) class of STT where gravity is characterized by the metric

$g_{\mu\nu}$  and the scalar field  $\phi$ ; the action is

$$S = \int d^4x \sqrt{g} \{ f(\phi) \mathcal{R}[g] + h(\phi) g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - 2U(\phi) + L_m \}. \quad (1)$$

Here  $\mathcal{R}[g]$  is the scalar curvature,  $g = |\det(g_{\mu\nu})|$ ;  $f$ ,  $h$  and  $U$  are certain functions of  $\phi$ , varying from theory to theory,  $L_m$  is the matter Lagrangian.

This formulation of the theory corresponds to the Jordan conformal frame, in which matter particles move along geodesics and hence the weak equivalence principle is valid, and non-gravitational fundamental constants do not change. In other words, this is the frame well describing the existing laboratory, geophysical and cosmological observations.

Among the three functions of  $\phi$  entering into (1) only two are independent since there is a freedom of transformations  $\phi = \phi(\phi_{\text{new}})$ . We use this arbitrariness, choosing  $h(\phi) \equiv 1$ , as is done, e.g., in Ref. [11]. Another standard parametrization is to put  $f(\phi) = \phi$  and  $h(\phi) = \omega(\phi)/\phi$  (the Brans-Dicke parametrization of the general theory (1)). In our parametrization  $h \equiv 1$ , the Brans-Dicke function  $\omega(\phi)$  is  $\omega(\phi) = f/f_\phi^2$ ; here and henceforth, the subscript  $\phi$  denotes a derivative with respect to  $\phi$ . The Brans-Dicke STT is the particular case  $\omega = \text{const}$ , so that in (1)

$$f(\phi) = \phi^2/(4\omega), \quad h \equiv 1. \quad (2)$$

The field equations that follow from (1) read

$$\square \phi - \frac{1}{2} \mathcal{R} f_\phi + U_\phi = 0, \quad (3)$$

$$f(\phi) \left( \mathcal{R}_\mu^\nu - \frac{1}{2} \delta_\mu^\nu \mathcal{R} \right) = -\phi_{,\mu} \phi^{,\nu} + \frac{1}{2} \delta_\mu^\nu \phi^{,\alpha} \phi_{,\alpha} - \delta_\mu^\nu U(\phi) + (\nabla_\mu \nabla^\nu - \delta_\mu^\nu \square) f - T_{\mu(m)}^\nu, \quad (4)$$

where  $\square$  is the D'Alembert operator, and the last term in (4) is the energy-momentum tensor of matter.

Consider now isotropic cosmological models with the standard FRW metric

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right], \quad (5)$$

where  $a(t)$  is the scale factor of the Universe, and  $k = 1, 0, -1$  for closed, spatially flat and hyperbolic models, respectively. Accordingly, we assume  $\phi = \phi(t)$  and the energy-momentum tensor of matter in the perfect fluid form  $T_{\mu(m)}^\nu = \text{diag}(\rho, -p, -p, -p)$  ( $\rho$  is the density and  $p$  is the pressure).

The field equations in this case can be written as follows (the dot denotes  $d/dt$ ):

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{3}{a^2}(a\ddot{a} + \dot{a}^2 + k) + U_\phi = 0, \quad (6)$$

$$\frac{3f}{a^2}(\dot{a}^2 + k) = \frac{1}{2}\dot{\phi}^2 + U - 3\frac{\dot{a}}{a}\dot{f} + \rho, \quad (7)$$

$$\frac{f}{a^2}(2a\ddot{a} + \dot{a}^2 + k) = -\frac{1}{2}\dot{\phi}^2 + U - \ddot{f} - 2\frac{\dot{a}}{a}\dot{f} - p. \quad (8)$$

To connect these equations with observations, let us fix the cosmic time  $t$  at the present epoch (i.e., consider the instantaneous values of all quantities) and introduce the standard observables:

$$H = \dot{a}/a \text{ (the Hubble parameter),}$$

$$q = -a\ddot{a}/\dot{a}^2 \text{ (the deceleration parameter),}$$

$$\Omega_m = \rho/\rho_{\text{cr}} \text{ (the matter density parameter),}$$

where  $\rho_{\text{cr}}$  is the critical density, or, in our model, the r.h.s. of Eq. (7) in case  $k = 0$ :  $\rho_{\text{cr}} = 3fH^2$ . This is slightly different from the usual definition  $\rho_{\text{cr}} = 3H^2/8\pi G$  where  $G$  is the Newtonian gravitational constant. The point is that the locally measured Newtonian constant in STT differs from  $1/(8\pi f)$ ; provided the derivatives  $U_{\phi\phi}$  and  $f_{\phi\phi}$  are sufficiently small, one has [11]

$$8\pi G_{\text{eff}} = \frac{1}{f} \frac{2\omega + 4}{2\omega + 3}. \quad (9)$$

(more details can be found in Refs. [14, 15] where the connection between  $G_{\text{eff}}$  and  $\omega$  was studied on the basis of cosmological solutions with local inhomogeneities and the equations of particle motion.)

Since, according to the solar-system experiments,  $\omega \geq 2500$ , for our order-of-magnitude reasoning we can safely put  $8\pi G = 1/f$ , and, in particular, our definition of  $\rho_{\text{cr}}$  now coincides with the standard one.

The time variation of  $G$ , to a good approximation, is

$$\dot{G}/G \approx -\dot{f}/f = gH, \quad (10)$$

where, for convenience, we have introduced the coefficient  $g$  expressing  $\dot{G}/G$  in terms of the Hubble parameter  $H$ .

Eqs. (6)–(8) contain too many arbitrary parameters for making a good estimate of  $g$ . Let us now introduce some restrictions according to the current state of observational cosmology:

(i)  $k = 0$  (a spatially flat cosmological model, so that the total density of matter equals  $\rho_{\text{cr}}$ );

(ii)  $p = 0$  (the pressure of ordinary matter is negligible compared to the energy density);

(iii)  $\rho = 0.3\rho_{\text{cr}}$  (the ordinary matter, including its dark component, contributes to only 0.3 of the critical density; unusual matter, which is here represented by the scalar field, comprises the remaining 70 per cent).

Then Eqs. (7) and (8) can be rewritten in the form

$$\frac{1}{2}\dot{\phi}^2 + U - 3H\dot{f} = 2.1H^2f, \quad (11)$$

$$-\frac{1}{2}\dot{\phi}^2 + U - 2H\dot{f} - \ddot{f} = (1 - 2q)H^2f. \quad (12)$$

Subtracting (12) from (11), we exclude the “cosmological constant”  $U$ , which can be quite large but whose precise value is hard to estimate. We obtain

$$\dot{\phi}^2 - H\dot{f} + \ddot{f} = (1.1 + 2q)H^2f. \quad (13)$$

The first term in Eq. (13) can be represented in the form

$$\dot{\phi}^2 = \dot{f}^2(df/d\phi)^{-2} = \dot{f}^2\omega/f,$$

and  $\dot{f}/f$  can be replaced with  $-gH$ . The term  $\ddot{f}$  can be neglected for our estimation purposes. To see this, let us use as an example the Brans-Dicke theory, in which  $f = \phi^2/(4\omega)$ . We then have

$$\ddot{f} = (\dot{\phi}^2 + \phi\ddot{\phi})/(2\omega);$$

here the first term is the same as the first term in Eq. (13), times the small parameter  $1/(2\omega)$ . Assuming that  $\phi\ddot{\phi}$  is of the same order of magnitude as  $\dot{\phi}^2$  (or only slightly greater), we see that, generically,  $|\ddot{f}| \ll \dot{\phi}^2$ . Note that our consideration is not restricted to the Brans-Dicke theory and concerns the model (1) with an arbitrary function  $f(\phi)$  and an arbitrary potential  $U(\phi)$ .

Neglecting  $\ddot{f}$ , we see that (13), divided by  $H^2f$ , leads to a quadratic equation with respect to  $g$ :

$$\omega g^2 + g - q' = 0, \quad (14)$$

where  $q' = 1.1 + 2q$ .

According to modern observations, the Universe is expanding with an acceleration, so that the parameter  $q$  is, roughly,  $-0.5 \pm 0.2$ , hence we can take  $|q'| \leq 0.4$ . (Note that this condition is only plausible rather than certain.)

In case  $q' = 0$  we simply obtain  $g = -1/\omega$ . Assuming

$$H = h_{100} \cdot 100 \text{ km}/(\text{s.Mpc}) \approx h_{100} \cdot 10^{-10} \text{ yr}^{-1}$$

and  $\omega \geq 2500$ , we come to the estimate

$$|\dot{G}/G| \leq 4 \cdot 10^{-14} h_{100} \text{ yr}^{-1}, \quad (15)$$

where  $h_{100}$  is, by modern views, close to 0.7. So (15) becomes

$$|\dot{G}/G| \leq 3 \cdot 10^{-14} \text{ yr}^{-1}. \quad (16)$$

For nonzero values of  $q'$ , solving the quadratic equation (14) and assuming  $q'\omega \gg 1$ , we arrive at the estimate  $|g| \sim \sqrt{q'/\omega}$ , so that, taking  $q' = 0.4$  and again  $\omega \geq 2500$ , we have instead of (15)

$$|\dot{G}/G| \leq 1.3 \cdot 10^{-12} h_{100} \text{ yr}^{-1} \approx 0.9 \cdot 10^{-12} \text{ yr}^{-1}, \quad (17)$$

where we have again put  $h_{100} = 0.7$ .

We conclude that, in the framework of the general STT, modern cosmological observations, taking into account the solar-system data, restrict the possible variation of  $G$  to values within  $10^{-12}/\text{yr}$ . This estimate may be considerably tightened if the matter density parameter  $\Omega_m$  and the (negative) deceleration parameter  $q$  will be determined more precisely.

Our estimates are rather universal since they do not use special solutions to the field equations, but actually rest on the well-justified assumption that the expansion of the Universe occurs without abrupt changes in its parameters during a fairly long period before now.

### 3. Discussion

Summarizing the above considerations, we can conclude that restrictions on possible nonzero values of  $\dot{G}$  give no bound on the possible class of generalized gravitation theories, but in the framework of some fixed theory any restriction on  $\dot{G}$  restricts the possible class of models.

We note that similar estimations of  $\dot{G}$  can be made for different multicomponent multidimensional models [13], giving a result on the level of  $10^{-12}/\text{yr}$  and less for, e.g., dust and p-brane matter sources.

We can also mention that the behaviour of the gravitational constant can actually be much more complex and intriguing than a simple time (or even range) dependence: very recently, there appeared two papers, which may open a new series of theoretical and experimental studies related to possible anisotropy in the absolute value of  $G$  [16] and/or its possible dependence on the latitude and longitude of the laboratory where  $G$  was measured [17].

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